# Split Dominating Set of an Interval Graph Using an Algorithm.

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**Abstract** : We study the problem of computing minimum dominating sets of n intervals on lines. Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, ecology, biology, computer sciences and particularly useful in cyclic scheduling and computers storage allocation problems etc. In this paper we discussed the notions new algorithms for split domination in graphs using (minimum dominating set) MDS algorithm. We get many bounds and split domination number.

**Key Words** : Interval family, Interval graph, Connected graph, Dominating Set, split dominating set, Connected dominating set, split dominating number.

## 1 INTRODUCTION

We consider the problem of incrementally computing a minimal dominating set of an interval graph after the insertion or deletion of a set of lines. Let

$$I = \{I_1, I_2, ...., I_n\}$$

be the given interval family. Each interval i in I is represented by $[a_i, b_i]$, for i = 1, 2 ,..., n. Here $a_i$ is called the left endpoint and $b_i$ the right endpoint of the interval $I_i$. Without loss of generality we may assume that all end points of the intervals in I which are distinct between 1and 2n. The intervals are labelled in the increasing order of their right endpoints. Two intervals i and j are said to intersect each other, if they have non-empty intersection. A graph $G = (V, E$ is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. That is, if $i = [a_i, b_i]$ and $j = [a_j, b_j]$, then i and j intersect means either $a_j < b_i$ are $a_i < b_j$. Kulli .V.R et.all [1] introduced the concept of split and non split domination in graphs and also in Maheswari, B et all[2] . A non empty set $D \subseteq V$ of a graph G is a dominating set[3] of G if every vertex in $\langle V - D \rangle$ is adjacent to some vertex in D. The domination number $\Upsilon(G)$ is the minimum cardinality taken over all the minimal dominating sets of G. A dominating set is said to be minimum dominating set if it's domination number is minimum. A graph G is said to be connected, if there is a path between any two vertices of G. Otherwise it is disconnected. . Let G be a connected graph . If V is a vertex set of G such that G -V is disconnected then the vertex v is called a cut vertex. A dominating set D of G is a connected dominating set [4] if the induced sub graph $\langle D \rangle$ is connected. A dominating set $D \subseteq V$ of a graph G is a split dominating set (SDS) if the induced subgraph $\langle V - D \rangle$ is disconnected. The split domination number is the minimum cardinality of a split dominating set. It is denoted by $\Upsilon_s(G)$. The neighbourhood of a vertex v∈V is set consisting all vertices adjacent to v (including v). It is denoted by $nbd[v]$.Let $nbd[i]$ be defined as the set of vertices adjacent to i including i. Let max(i) denotes the largest interval in $nbd[i]$. Guruprakash. C. D.,Mallikarjuna Swamy.B. P [5], Minimum Matching Dominating Sets and its Apllications in Wireless Networks define Next(i) = j if and only if $b_i < a_j$ and there does not exist an interval k such that $b_i < a_k < a_j$ . If there is no such j, we define Next i = null.

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## 2 ALGORITHM OF MDS

**Input :** Let $I = \{1, 2, ....., n\}$ be Interval family

**Output:** Minimum dominating set of an interval graph G.

Step 1 : Let MDS = {max(1)}.

Step 2 : LI = The largest interval in MDS.

Step 3 : Compute Next LI .

Step 4 : If Next LI = null the goto step 8.

Step 5 : Find max Next LI .

Step 6 : If max(Next(LI)) does not exist then max(Next(LI)) = Next(LI).

ISSN 2229-5518

Step 7 : MDS = MDS $\cup$ max Next LI goto step 2.

Step 8 : End.

## 3 AN ALGORITHM FOR FINDING SPLIT DOMINATING SET.

**Input:** Interval family $I = \{1, 2, 3, \ldots, n\}$

**Output**: Split dominating set and induced sub graph of an interval graph G is disconnected.

Step 1: Set V = U i and U i = i +1 where i = 0 to n −1.

Step 2 : Find MDS = D i for some i where 0 < i < n −1.

Step 3 : $\Upsilon = |MDS|$.

Step 4 : V − MDS = S i , where i = 0 to $n - \Upsilon - 1$

Step 5 : for i = 0 to $n - \Upsilon - 1$

{

If max S i = n then

$p = \max S[i] - \Upsilon - 1$

else

If max S i = n-1 then

$p = \max S[i] - \Upsilon - 2$

( p values depends on $\Upsilon$)

else

$p = \max S[i] - 1$

for j = i +1 to p

{

If S i , S j $\in$ E in G

Plot a line from S[i] to S[j]

}

}

The induced sub graph $G_1$ obtained.

Step 6 : if w $G_1$ > w G

$G_1$ is disconnected then go to step 7

Step 7 : $S_1$ = max S 0 in $G_1$.

Step 8 : $LI_1$ = The largest interval in $S_1$.

Step 9 : Compute Next $LI_1$ in $G_1$.

Step 10 : If Next $LI_1$ = null the go to step 14.

Step 11 : Find max Next $LI_1$ in $G_1$

Step 12 : If max (Next($LI_1$)) does not exist then max (Next($LI_1$)) = Next($LI_1$) in $G_1$.

Step 13 : $S_1$ = $\{S_1\} \cup \{$max (Next($LI_1$))$\}$ go to step 8.

Step 14 : End.

## 4 MAIN THEOREMS

**Theorem 4. 1 :** Let $I = \{I_1, I_2, \ldots, I_n\}$ be an interval family and G is an interval graph corresponding to I. If i and j are any two intervals in I such that $i \in D$, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and there is no interval $k \neq i$ to the right of j that intersects j, then split domination occurs in G.

**Proof**: Let $I = \{I_1, I_2, \ldots, I_n\}$ be the given interval family and G it's corresponding interval graph . Let i and $j \neq 1$ be any two intervals in I such that j is contained in i. Let D be a dominating set of G and if $i \in D$. Let m be an interval in I which is to the left of j and intersect j. Further by our assumption there is no interval k > j such that k intersect j. Now j is contained in i implies j < i. Therefore j is not adjacent to any vertex in the set $\{i+1, i+2, i+3, \ldots, n\}$ and $i \in D$ implies that the induced subgraph $\langle V - D \rangle$ does not contain i. This implies that there is a disconnection at j. Hence we have split domination in G. Our aim to show that an algorithm to find split dominating set of an interval graph with an illustration.

**ILLUSTRATTION**

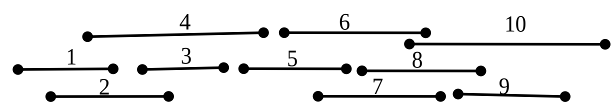


**Figure 1. Interval family**

We construct an interval graph G from an interval family $I = \{1, 2, 3, \ldots, 10\}$ as follows.

nbd 1 = 1,2,4 , nbd 2 = 1,2,3,4 , nbd 3 = 2,3,4 ,

nbd 4 = 1,2,3,4,5 , nbd 5 = 4,5,6,7 ,

nbd 6 = 5,6,7,8,10 , nbd 7 = 5,6,7,8,10 ,

nbd 8 = 6,7,8,9,10 , nbd 9 = 8,9,10 ,

nbd 10 = 6,7,8,9,10 .

max 1 = 4, max 2 = 4, max 3 = 4, max 4 = 5,

max 5 = 7, max 6 = 10, max 7 = 10, max 8 = 10,

max 9 = 10, max 10 = 10 .

Next(1) = 3, Next(2) = 5, Next(3) = 5,

Next(4) = 6, Next(5) = 8, Next(6) = 9,

Next(7) = 9, Next(8) = null, Next(9) = null,

Next(10) = null.

**Procedure For MDS**

**Input:** Interval family given in fig.1

Step 1 : MDS = 4

Step 2 : LI = 4.

Step 3 : Next(4) = 6.

Step 4 : max(6) = 10.

Step 5 : MDS = $\{4\} \cup \{10\}$ = 4,10 goto step2.

Step 6 : LI = 10.

Step 7 : Next(10) = null.

Step 8 : End.

Out put : {4,10} is the minimum dominating set of an interval graph G.

**Procedure For SDS**

Step 1: $V = \{1, 2, \ldots 10\}$.

Step 2: $MDS = \{4,10\}$.

Step 3: $\Upsilon = 2$.

Step 4: $S = \{1,2,3,5,6,7,8,9\}$.

Step 4: for $i = 0$ then

$\max S[0] = 4 \Rightarrow p = 4 - 1 = 3$.

$\Rightarrow j = 1$ to 3.

$(S[0]S[1]) = (1,2) \in E$ in G, plot a line from 1 to 2

$(S[0], S[2]) = (1,3) \notin E$ in G,

$(S[0], S[3]) = 1,5 \notin E$ in G

for $i = 1, \max S[1] = 4 \Rightarrow p = 4 - 1 = 3$.

$\Rightarrow j = 2$ to 3.

$(S[1], S[2]) = (2,3) \in E$ in G, plot a line from 2 to 3.

$(S[1], S[3]) = (2,5) \notin E$ in G.

for $i = 2$, $\max S[2] = 4 \Rightarrow p = 4 - 1 = 3$.

$\Rightarrow j = 3$ to 3.

$(S[2], S[3]) = (3,5) \notin E$ in G.

for $i = 3$, $\max S[3] = 7 \Rightarrow p = 7 - 1 = 6$

$\Rightarrow j = 4$ to 6

$(S[3], S[4]) = (5,6) \in E$ in G, plot a line from 5 to 6.

$(S[3], S[5]) = (5,7) \in E$ in G, plot a line from 5 to 7.

$(S[3], S[6]) = (5,8) \notin E$ in G

for $i = 4$, $\max S\ 4 = 10 \Rightarrow p = 10 - 2 - 1 = 7$

$\Rightarrow j = 5$ to 7

$(S[4], S[5]) = (6,7) \in E$ in G, plot a line from 6 to 7.

$(S[4], S[6]) = (6,8) \in E$ in G, plot a line from 6 to 8.

$(S[4], S[7]) = (6,9) \notin E$ in G.

for $i = 5, \max S[5] = 10 \Rightarrow p = 10 - 2 - 1 = 7$.

$\Rightarrow j = 6$ to 7.

$(S[5], S[6]) = (7,8) \in E$ in G, plot a line from 7 to 8.

$(S[5], S[7]) = (7,9) \notin E$ in G.

for $i = 6$, $\max S[6] = 10 \Rightarrow p = 10 - 2 - 1 = 7$.

$\Rightarrow j = 7$ to 7.

$(S[6], S[7]) = (8,9) \in E$ in G, plot a line from 8 to 9.

The induced sub graph $G_1$ = G-MDS is obtained.

Step 6: $w\ G_1 > w\ G \Rightarrow 2 > 1$

Therefore $G_1$ is disconnected, then go to step 7

Step 7 : $S_1 = \max S[0] = 2$.

Step 8: $LI_1 = 2$.

Step 9: Next(2) = 5.

Step 10: $max(5) = 7$.

Step 11: $S_1 = \{2\} \cup \{7\} = \{2,7\}$ goto step 8.

Step 12: $LI_1 = 7$.

Step 13: Next(7) = 9.

Step14: max(9) = 9

Step 15: $S_1 = \{2,7\} \cup \{9\} = \{2,7,9\}$ goto step 8.

Step 16: $LI_1 = 9$.

Step 17: Next(9) = null.

Step 18: End.

**Out put** : {4,10} is the split dominating set of the interval family as in fig.1, dominating set of $G_1$ is {2,7,9} and $\Upsilon_s < \Upsilon(G_1)$

**Theorem 4.2 :** Let D be a minimum dominating set of the given interval graph G. If i and j are any two intervals in I such that j is contained in i and if there is no other interval $k \neq i$ that intersects j the split domination occurs in G.

**Proof:** Let $I = \{I_1, I_2, \ldots, I_n\}$ be an interval family and G is an interval graph corresponding to I. Let i and j be any two intervals in I such that j is contained i. If there is no interval $k \neq i$ that intersect j. Then clearly i lies in the dominating set D. Further in induced sub graph $\langle V - D \rangle$ the vertex j is not adjacent to any other vertex and infact j becomes as isolated vertex in induced sub graph $\langle V - D \rangle$. Hence we get split domination.

**ILLUSTRATION**

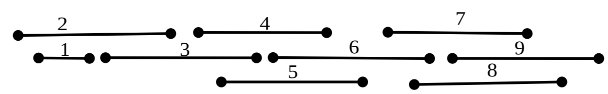


**Figure 2. Inteval Family**

As follows an algorithm with an illustration for neighbours as given interval family I.

We construct an interval graph G from an interval family $I = \{1, 2, \ldots, 9\}$ as follows

$nbd[1] = \{1,2\}$, $nbd[2] = \{1,2,3\}$, $nbd[3] = \{2,3,4,5\}$,

$nbd[4] = \{3,4,5,6\}$, $nbd[5] = \{3,4,5,6\}$, $nbd[6] = \{4,5,6,7,8\}$,

$nbd[7] = \{6,7,8,9\}$, $nbd[8] = \{6,7,8,9\}$, $nbd[9] = \{7,8,9\}$

$\max(1) = 2$, $max(2) = 3$, $\max(3) = 5$, $\max(4) = 6$,

$\max(5) = 6$, $\max(6) = 8$, $\max(7) = 9$, $\max(8) = 9$,

$\max(9) = 9$.

Next(1) = 3, Next(2) = 4, Next(3) = 6, Next(4) = 7,

Next(5) = 7, Next(6) = 9, Next(7) = null, Next(8) = null,

Next(9) = null.

**Procedure For MDS**

**Input:** Interval family given in fig.2

Step 1 : MDS = 2.

Step 2: LI = 2.

Step 3 : Next(2) = 4.

ISSN 2229-5518

Step 4 : $\max(4) = 6.$

Step 5 : $MDS = \{2\} \cup \{6\} = \ 2,6$ goto step2.

Step 6 : $LI = 6.$

Step 7 : $Next(6) = 9.$

Step 8 : $\max(9) = 9.$

Step 9 : $MDS = \{2,6\} \cup \{9\} = \ 2,6,9$ goto step 2.

Step 10 : $LI = 9.$

Step 11 : $Next(9) = null.$

Step 12 : End.

Out put : {2,6,9} is the minimum dominating set of an interval graph G.

**Procedure For SDS**

**Input** : Let Interval family $I = \{1,2,....,9\}$. Similarly the above method as follows nbd[i], max(i)and Next(i).

Step 1 : V={1,2,3,4,5,6,7,8,9}.

Step 2 : $MDS = \{2,6,9\}$ by using above MDS algorithm

Step 3 : $\Upsilon = 3.$

Step 4 : $S[\ ] = \{1,3,4,5,7,8\}$

Step 5 : for i = 0 then max $S[0] = 2 \Rightarrow p = 2 - 1 = 1.$

$\Rightarrow j = 1$ to 1.

$(S[0], S[1]) = (1,3) \notin E$ in G.

for i = 1 then max $S[1] = 4 \Rightarrow p = 5 - 1 = 4$

$\Rightarrow j =$ 2 to 4

$(S[1], S[2]) = (3,4) \in E$ in G , plot a line from3 to 4.

$(S[1], S[3]) = (3,5) \in E$ in G , plot a line from 3 to 5.

$(S[1], S[4]) = (3,7) \notin E$ in G.

for i = 2 , max $S[2] = 6 \Rightarrow p = 6 - 1 = 5.$

$\Rightarrow j = 3$ to 5.

$(S[2], S[3]) = (4,5) \in E$ in G, plot a line from 4 to 5

$(S[2], S[4]) = (4,7) \notin E$ in G,

$(S[2], S[5]) = (4,8) \notin E$ in G.

for i = 3, max $S[3] = 6 \Rightarrow p = 6 - 1 = 5.$

$\Rightarrow j = 4$ to 5.

$(S[3], S[4]) = (5,7) \notin E$ in G,

$(S[3],S[5]) = (5,8) \notin E$ in G.

for i = 4,max $S[4] = 9 \Rightarrow p = 9 - 3 - 1 = 5'$

$\Rightarrow j = 5$ to 5.

$(S[4],S[5]) = (7,8) \in E$ in G , plot a line from 7 to 8 .

The induced sub graph $G_1$=G-MDS obtained.

Step 6 : $w\ G_1\ > w\ G\ \Rightarrow 3 > 1.$

Therefore $G_1$ is disconnected then go to step 7

Step 7 : $S_1 = \ \max S\ 0\ = 1.$

Step 8 : $LI_1 = 1.$

Step 9 : $Next(1) = 3$ in $G_1$.

Step 10 : $\max(3) = 5$ in $G_1$.

Step 11: $S_1 = \{1\} \cup \{5\} = \{1,5\}$ goto step 8.

Step 12 : $LI_1 = 5.$

Step 13 : $Next(5) = 7$ in $G_1$.

Step 14 : $\max(7) = 8$ in $G_1$.

Step 15 : $S_1 = \{\{1,5\} \cup \{8\} = \{1,5,8\}$ goto step 8.

Step 16 : $LI_1 = 8$ in $G_1$.

Step 17 : $Next(8) = null$ in $G_1$.

Step 18 : End.

**Output** : {2,6,9} is the split dominating set of the interval family as in fig.2., dominating set of $G_1$ is (1,5,8} and $\Upsilon_s = \Upsilon(G_1)$.

**Theorem 4.3 :** Let $I = \{I_1, I_2, ...., I_n\}$ be an interval family and D is a minimum dominating set of the given interval graph G. If i, j, k are any three consecutive intervals such that $i < j < k$ and if $j \in D$, and i intersects j, j intersect k and i does not intersect k then split domination occurs in G.

**Proof :** Suppose $I = \{I_1, I_2, ...., I_n\}$ be an interval family . If i, j, k be three consecutive intervals such that $i < j < k$ and i intersect j, j intersect k, but i does not intersect k. Suppose $j \in D$, where D is a minimum dominating set. Then i and k are not adjacent in $\langle V - D \rangle$ . That is there is a disconnection between i and k provided, there is no $m \in I$ , $m < k$ such that m intersects k. If such an m exists, then since $m < k$ we must have $m < i < j < k$ . Now m intersects k implies i and j also intersect. This is a contradiction to hypothesis. So such a m does not exist hence we get split domination . As usual as follows an algorithm to find a split dominating set of an interval graph G.

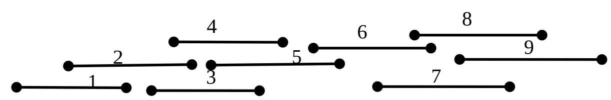


**Figure 3. Interval family**

We construct an interval graph from an interval family $I = \{1,2,....,9\}$ as follows

$nbd[1] = \{1,2\}$, $nbd[2] = \{1,2,3,4\}$, $nbd[3] = \{2,3,4,5\}$,

$nbd[4] = \{2,3,4,5\}$, $nbd[5] = \{3,4,5,6\}$, $nbd[6] = \{5,6,7,8\}$,

$nbd[7] = \{6,7,8,9\}$, $nbd[8] = \{6,7,8,9\}$, $nbd[9] = \{7,8,9\}$.

$\max(1) = 2$, $\max(2) = 4$, $\max(3) = 5$, $\max(4) = 5$, $\max(5) = 6$,

$\max(6) = 8$, $\max(7) = 9$ , $\max(8), \max(9) = 9.$

$Next(1) = 3, Next(2) = 5$, $Next(3) = 6, Next(4) = 6,$

$Next(5) = 7, Next(6) = 9, Next(7) = null,$

$Next(8) = null, Next(9) = null.$

**Procedure For MDS**

**Input:** Interval family given in fig. 3

Step 1 : $MDS = 2.$

ISSN 2229-5518

Step 2 : $LI = 2$.

Step 3 : Next (2) = 5.

Step 4 : max(5) = 6.

Step 5 : $MDS = \{2\} \cup \{6\} = \{2,6\}$ goto step2.

Step 6 : $LI = 6$.

Step 7 : Next(6) = 9.

Step 8 : max(9) = 9.

Step 9 : $MDS = \{2,6\} \cup \{9\} = \{2,6,9\}$ goto step 2.

Step 10 : $LI = 9$.

Step 11 : Next(9) = null.

Step 12 : End.

Out put : {2,6,9} is the minimum dominating set of an interval graph G.

**Procedure For SDS**

**Input**: Interval family $I = \{1,2,....,9\}$ as the above method follows nbd[i], max(i)and Next(i).

Step 1 : V={1,2,3,4,5,6,7,8,9}

Step 2 : $MDS = \{2,6,9\}$.

Step 3 : $\Upsilon = 3$.

Step 4 : $S[\,] = \{1,3,4,5,7,8\}$.

Step 5 : for i = 0 then max S[0] = $2 \Rightarrow p = 2-1 = 1$.

$\Rightarrow j = 1$ to 1.

$(S[0], S[1]) = (1,3) \notin E$ in G.

for i = 1 then max S[1] = $5 \Rightarrow p = 5-1 = 4$.

$\Rightarrow j = 2$ to 4.

$(S[1], S[2]) = (3,4) \in E$ in G , plot a line from 3 to 4.

$(S[1], S[3]) = (3,5) \in E$ in G, plot a line from 3 to 5.

$(S[1], S[4]) = (3,7) \notin E$ in G.

for i = 2,max S[2] = $5 \Rightarrow p = 5-1 = 4$.

$\Rightarrow j = 3$ to 4.

$(S[2], S[3]) = (4,5) \in E$ in G, plot a line from 4 to 5.

$(S[2], S[4]) = (4,7) \notin E$ in G.

for i = 3, max S[3] = $6 \Rightarrow p = 6-1 = 5$.

$\Rightarrow j = 4$ to 5.

$(S[3], S[4]) = (5,7) \notin E$ in G, $(S[3],S[5]) = (5,8) \notin E$ in G.

for i = 4,max S[4] = $9 \Rightarrow p = 9-3-1 = 5$

$\Rightarrow j = 5$ to 5

$(S[4], S[5]) = (7,8) \in E$ in G, plot a line from7 to 8.

The induced sub graph $G_1$=G-MDS obtained.

Step6 : $w(G_1) > w(G) \Rightarrow 3 > 1$

Therefore $G_1$ is disconnected then go to step 7.

Step 7 : $S_1 = \{\max S[0]\} = 1$.

Step 8 : $LI_1 = 1$.

Step 9 : Next(1) = 3 in $G_1$.

Step 10 : max(3) = 5 in $G_1$.

Step 11 : $S_1 = \{1\} \cup \{5\} = \{1,5\}$ goto step 8.

Step 12 : $LI_1 = 5$.

Step 13 : Next(5) = 7 in $G_1$.

Step 14 : max(7) = 8 in $G_1$.

Step 15 : $S_1 = \{1,5\} \cup \{8\} = \{1,5,8\}$ goto step 8.

Step 16 : $LI_1 = 8$ in $G_1$.

Step 17 : Next(8) = null in $G_1$.

Step 18 : End.

**Output** : {2,6,9} is the split dominating set of the interval family as in fig.3. , dominating set of $G_1$ is (1,5,8} and $\Upsilon_s = \Upsilon(G_1)$ and $\Upsilon_s = \Upsilon(G_1)$

## 5 CONCLUSIONS

In this paper we discussed the split dominating set of an interval graph using an algorithm. We present an algorithm approximation scheme for this problem on an interval graph corresponding to an interval family with bounded growth. The scheme is robust and Thus returns for any undirected graph given as input a meaningful output. For which an optimal, partial solution can be obtained. While this approach is already used for related problems, feasibility when combining the partial solutions is an issue for sets that have to remain both dominating set and split dominating set of an interval graph. We solve this issue by a disconnected processing repair algorithm.

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